EVALUATING FATIGUE PRODUCING VIBRATION ENVIRONMENTS USING THE SHOCK RESPONSE SPECTRUM

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ABSTRACT

The shock response spectra computed directly from the time histories of stationary vibration data are compared to the expected values for the maximum response peaks computed from the autospectra of the random data with a Gaussian probability density function. The results reveal excellent agreement for the case where the vibration data are truly random. However, for the complex vibration environment generated by a repetitive shock machine, the shock response spectra computed directly from the time history data are higher than the expected values for the maximum response peaks computed from the autospectra of the random data. This indicates that, for the same autospectrum, the damage potential of a repetitive shock machine is greater than that for a truly random vibration. A quantitative example is presented using a Damage Potential Spectrum tool. The same conclusion undoubtedly applies to many other complex but not random vibration environments such as those produced by reciprocating engines.

INTRODUCTION

The shock response spectrum (SRS), which is broadly defined as the peak response of a simple oscillator (single degree-of-freedom) to an excitation as a function of the natural frequency of the oscillator [1], was originally introduced to evaluate the damage potential of mechanical transients. However, the SRS can also be used to evaluate the damage potential of stationary random vibrations as measured by the peak value for the response of a simple oscillator exposed to the vibration [2, 3] over a finite duration. This latter application of the SRS directly competes with the use of statistical procedures to predict the peak value for the response of a simple oscillator under the assumption that the excitation to the oscillator is a stationary random process [2 . 4]. Since the SRS does not require the excitation to be random, a direct comparison of SRS results to a statistical computation of the maximum value of the oscillator response can be used to evaluate the randomness of the excitation. Such a comparison is used in [5] to detect transients in otherwise stationary random vibration signals. Of interest here is the use of such a comparison to detect differences between the damage potential of random vibrations versus the complex (sometimes called quasi-random) vibrations produced by pneumatic hammer type vibration test machines, commonly referred to as repetitive shock machines.

STATISTICAL COMPUTATION OF SHOCK RESPONSE SPECTRUM

The computation of the shock response spectrum (SRS) for a stationary random vibration involves the determination of the maximum value for the response of a lightly damped, linear oscillator to the excitation, as illustrated in Figure 1(a). Assuming an acceleration excitation x(t) produces an acceleration response y(t), the frequency response function of the simple oscillator is given by [6]

$$H_{xy}(f) = \sqrt{\frac{1 + j2\zeta f_n}{1 - (f/f_n)^2 + j2\zeta f_n}} \qquad (1)$$

where f_n is the undamped natural frequency and ζ is the damping ratio of the oscillator. Assuming the acceleration excitation is random with an autospectrum $G_{xx}(f)$ that is reasonably uniform over the frequency range where f is near f_n, the response of the oscillator will be random with a narrow bandwidth, as shown in Figure 1(b), and will have a standard deviation approximated by [6]

$$\sigma_y = \sqrt{\frac{G_{xx}(f_n)\pi f_n\left[1 + 4\zeta^2\right]}{4\zeta}} \qquad (2)$$

It should be mentioned that in [2, 3], the standard deviation of the oscillator response is computed in terms of velocity, rather than acceleration, because the stress produced by the resonant response of a structure is proportional to velocity [7]. For the application at hand, however, only comparisons of relative values are of interest, so acceleration units are used since they are more familiar to most shock and vibration engineers.

It is recommended in [2, 3] that a conservative maximum value for the response of the oscillator to a stationary random excitation be estimated by

$$Y_m[P(T)] = \sigma_y \sqrt{2 \ln\left[\frac{f_n T}{P(T)}\right]} \quad ; \quad P(T) << 1 \qquad (3)$$

where

f_n = undamped natural frequency of the oscillator
T = duration of the excitation
σ = standard deviation of the oscillator response, as defined in Eq. (2)
P(T) = probability that the value Y_m will be exceeded during the exposure duration T.

For design purposes, a probability of P(T) = 0.05 (5%) is commonly assumed in Eq. (3) [2, 3]. However, to estimate an SRS, the expected value of Y_m is needed since this corresponds to the average value of the SRS as normally computed. The expected value for the maximum response of the oscillator can be estimated by [4]

$$E[Y_m] = \sigma_y \left| \sqrt{2 \ln(f_n T)} + 0.5772 / \sqrt{2 \ln(f_n T)} \right| \quad ; \quad \ln(f_n T) >> 1 \qquad (4)$$

where all terms are as defined in Eq. (3). The results in Eqs. (3) and (4) involve two critical assumptions, as follows:

1. The response y(t) of the oscillator has a normal (Gaussian) probability density function. As long as the excitation x(t) is random and the oscillator response is linear, even if x(t) is not Gaussian, this assumption is generally acceptable because the narrow bandwidth filtering of the oscillator suppresses deviations from the Gaussian form in the response y(t) [8].

2. The peak values of the oscillator response are statistically independent. It is clear from the relatively smooth variations in the envelope for the peak values of the oscillator response in Figure 1(b) that the peak values are not statistically independent, but computer simulation

studies reported in [2] indicate the statistical independence assumption is acceptable for values of $Y_m/\sigma_y > 3.5$, which corresponds to $fT > 250$ for a Gaussian random process.

COMPARISONS OF ESTIMATED AND COMPUTED RESULTS

Computations were performed on data produced by two sources, namely, (a) a computer generated random signal simulating a Gaussian random vibration, and (b) the signal from an accelerometer mounted on the table of a commercial repetitive shock (RS) machine. In both cases, an autospectrum (PSD) was computed using a conventional fast Fourier transform (FFT) based PSD analysis algorithm [6], and a shock response spectrum (SRS) was computed using the 'Ramp-Invariant Method'[9]. All analyses were performed over $T = 6.5$ seconds of data that were digitized using a sampling rate of 25,000 samples per second. The lower frequency limit for the analyses was fixed to 40 Hz to comply with the second assumption after Eq. (4). The upper frequency limit was fixed at *2,500* Hz (10% of the sampling rate) to restrict the magnitude error in the SRS values to less than *5%* [10], as well as to suppress an inherent bias error in the Ramp-Invariant Method [11]. Damping ratios of 5% ($\zeta = 0.05$ corresponding to Q = 10) and 1% ($\zeta = 0.01$ corresponding to Q = 50) were used for all standard deviation and SRS computations. The frequency resolution for the analyses was 10 Hz for the PSD values and 1/12 octave band for the SRS computations. However, all PSD and SRS results are presented at 1/3-octave band center frequencies for clarity.

Random Data

The simulated random vibration data were generated with a PSD of $G_{xx}(f)) = 0.003\ g^2/Hz$ over the frequency range from 40 to 2,500 Hz. The directly computed SRS for the simulated random vibration data, the expected value for the maximum response given by Eq. (4), and the $\zeta(T) = 0.05$ value given by Eq. (3), all computed with *5%* damping, are compared in Figure 2. Note in Figure 2 that the directly computed SRS values are in good agreement, on average, with the predicted values of Eq. (4), and are just enveloped by the P(T) = 0.05 values of Eq. (3), as would be expected with 19 SRS values. It is clear from these results that Eqs. (3) and (4), using the standard deviation computed from Eq. (2), provide accurate results for truly random data.

RS Machine Data

The PSD values of the vibration generated by the RS machine at the 1/3-octave band center frequencies used for the computations in Eqs. (2) through (4) are shown in Figure 3. The directly computed SRS values for the RS machine vibration data, the expected value of the maximum response given by Eq. (4), and the P(T) = 0.05 value given by Eq. (3), all computed with 5% damping, are compared in Figure 4. Note in Figure 4 that the directly computed SRS values exceed the predicted values of Eqs. (3) and (4) at most frequencies by margins of up to 3:1. It is clear from these results that Eqs. (3) and (4), using the standard deviation computed from Eq. (2) with 5% damping, do not provide accurate results for RS machine vibration data. Specifically, the RS machine is producing a higher ratio of maximum to standard deviation response values (Y_m/σ_y) than would be produced by a random response with a Gaussian probability density function. These results are consistent with the findings in [12], and further would be intuitively anticipated for a vibration response that is produced by a sequence of transients rather than a stationary random vibration.

The various SRS values for the RS machine computed with 1% damping are compared in Figure 5. It is seen in Figure 5 that the directly computed SRS values still exceed the predicted values of Eqs. (3) and (4) at most frequencies, but by much smaller margins than for the 5% damping results in Figure 4. This suggests that the discrepancies between the directly and indirectly computed SRS values for the RS machine vibration diminish with decreasing damping. This result might be explained as follows. Repetitive shock machines produce what is essentially a complex periodic vibration with natural and sometimes intentionally introduced random modulations of both magnitude and frequency. Hence, the vibration does have a limited random character. The half-power point bandwidth for a simple oscillator is approximated by $B_{hp} \approx 2\zeta f_n$ [6], that is, the bandwidth of the oscillator is directly proportional to the damping ratio. The narrow bandwidth filtering operation of the simple oscillators essentially invokes the Central Limit Theorem and thus suppresses deviations from the Gaussian form, where the narrower the bandwidth, the greater the suppression of non-Gaussian characteristics [8]. It follows that the narrower bandwidth for the 1% damping produces a more Gaussian response that makes Eqs. (2) through (4) more accurate. This conclusion is consistent with the findings in [13] that show the response of lightly damped cantilever beams exposed to the vibration produced by an RS machine is quite close to the Gaussian form, at least at natural frequencies above about 500 Hz.

CONCLUSIONS

The shock response spectra computed directly from the time histories of vibration data are compared to the expected value for the maximum response peak computed from the autospectra of the data under the assumption the data are random with a Gaussian probability density function. The results reveal excellent agreement for simulated vibration data that are in fact random in character. However, for the complex vibration environment generated by a repetitive shock machine, the shock response spectrum values computed directly from the time history data with *5%* damping (a common damping ratio for many structural assembles) are substantially higher (by up to 3:1) than the expected value for the maximum response peak computed from the autospectra of the data with *5%* damping under the assumption that the data are random. The discrepancies between the directly and indirectly computed shock response spectra are smaller when computed with 1% damping, but the directly computed results still exceed the indirectly computed results at most frequencies. This indicates that, for the same autospectrum, the damage potential of a repetitive shock machine is greater than that for a truly random vibration. This same conclusion undoubtedly applies to many other complex, but not truly random vibration environments, such as those produced by reciprocating engines.

REFERENCES

1. Rubin, S., "Concepts in Shock Data Analysis", Chapter 23 in *Shock and Vibration Handbook,* 5th edition, McGraw-Hill, New York, 1996.

2. Kern, D. L., et al, *Dynamic Environmental Criteria,* NASA-HDBK-7005, National Aeronautics and Space Administration, 2000.

3. Piersol, A. G., "Preliminary Design Procedures for Equipment Exposed to Random Vibration Environments", *Proc., 46th Annual Meeting of IEST,* Providence, RI, 2000.

4. Nigam, N. C., *Introduction to Random Vibrations,* MIT Press, Cambridge, MA, 1983.

5. Ahlin, K., "Detection of Transients in Vibration Time Histories", *Proc., 6th International Congress on Sound and Vibration,* Copenhagen, DK, 1999.

6. Bendat, J. S., and Piersol, A. *G., Random Data: Analysis and Measurement Procedures,* 3rd Edition, Wiley, NY, 2000.

7. Hunt, F. V., "Stress and Strain Limits on the Attainable Velocity in Mechanical Vibration", *J. Acoustical Society of America,* Vol. 32, No. 9, pp. 1123 .1128, 1960.

8. Papoulis, A., "Narrow-Band Systems and Gaussianity", *RADC-TR-71-225,* Rome Air Development Center, Griffis AFB, NY, 1971.

9. Smallwood, D. 0., "An Improved Recursive Formula for Calculating Shock Response Spectra", *Shock and Vibration Bulletin,* No. *56,* Part 1, pp. 279-287, 1986.

10. Wise, J. H., "The Effects of Digitizing Rate and Phase Distortion Errors on the Shock Response Spectrum", *Proc., 29th Annual Meeting of IES,* pp. 36-43, 1983.

11. Ahlin, K., "Shock Response Spectrum Calculation .An Improvement of the Smallwood Algorithm", *Proc., 70th Shock and Vibration Symposium,* Albuquerque, NM, 1999.

12. Henderson, G. R., "Dynamic Characteristics of Repetitive Shock Machines", *Proc., 39th Annual Technical Meeting of IES,* pp. 231-249, 1993.

13. Henderson, G. R., and Piersol, A. G., "Fatigue Damage Related Descriptor for Random Vibration Test Environments, *Sound & Vibration Magazine,* Vol. 29, No. 10, pp. 20-24, *1995.*

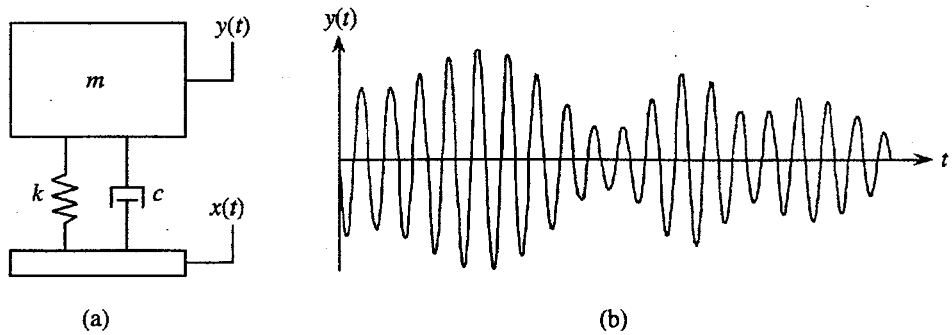

Figure 1. Illustration of a simple oscillator. (a) Skematic diagram. (b) Time history response to random excitation.

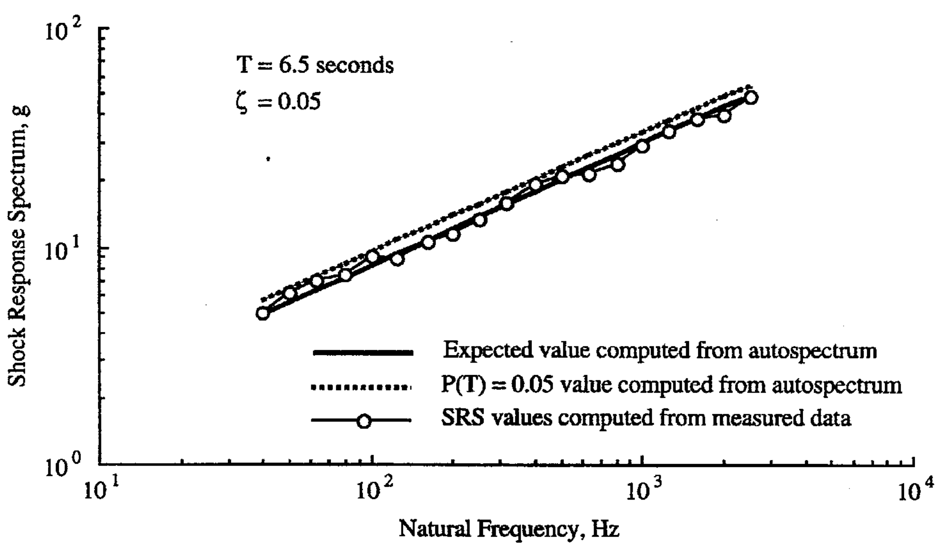

Figure 2. Shock response spectra for Gaussian random data computed with 5% damping.

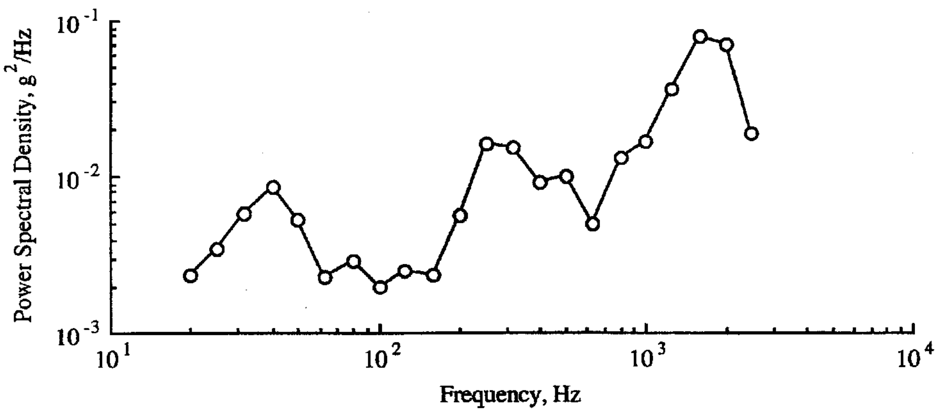

Figure 3. Autospectrum for Retetive Shock Machine Vibration.

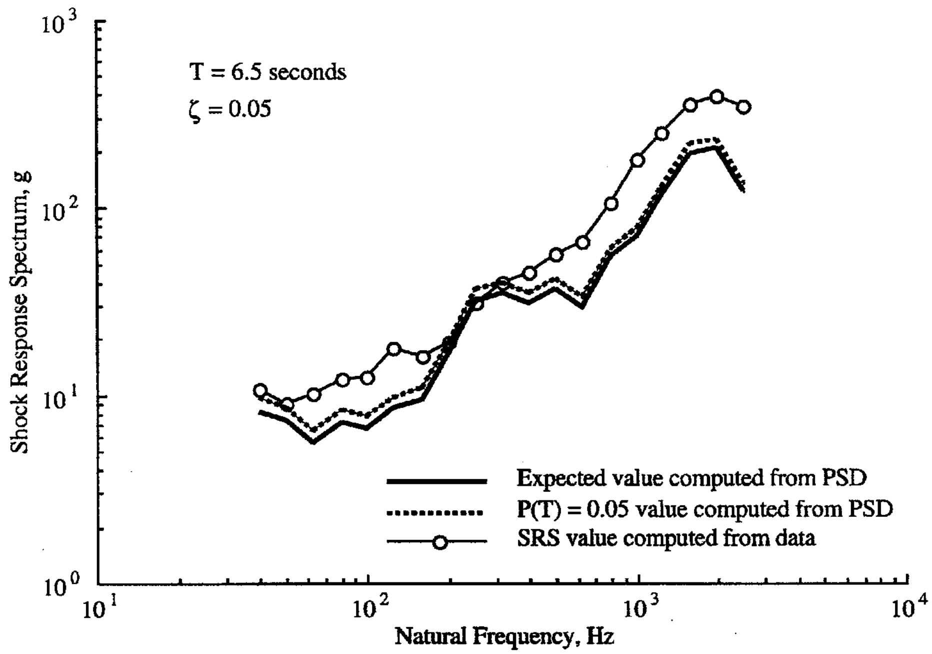

Figure 4. Shock response spectra for repetitive shock machine vibration computed with 5% damping.

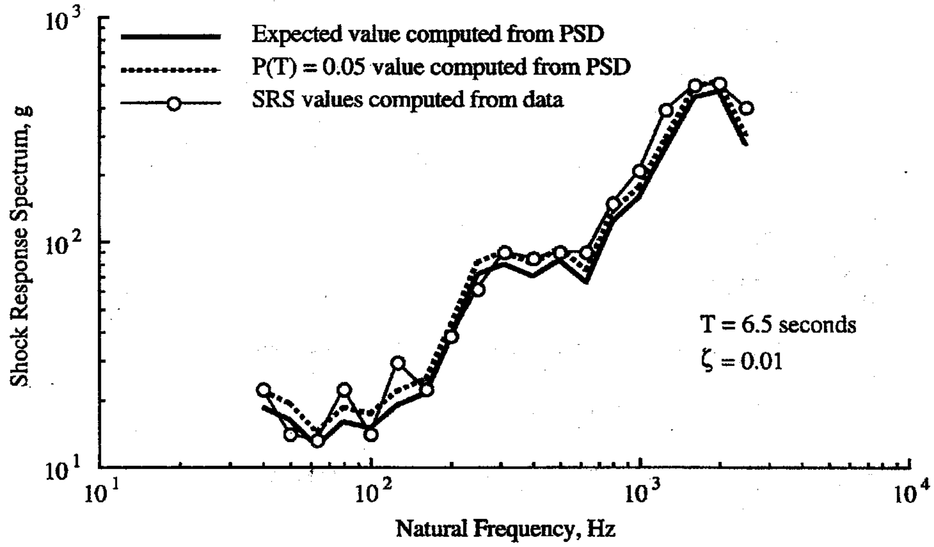

Figure 5. Shock response spectra for repetitive shock machine vibration computed with 1% damping.